

# INELASTIC NEUTRON SCATTERING, EPR AND SPIN CHIRALITY IN SPIN-FRUSTRATED $V_3$ and $Cu_3$ NANOMAGNETS WITH DZIALOSHINSKY-MORIYA EXCHANGE

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**Abstract.** The inelastic neutron scattering (INS) and EPR transitions are considered for the spin-frustrated  $V_3$  and  $Cu_3$  nanomagnets. It is shown that the DM exchange and distortions determine the Q-dependence and redistribution of the intensities of the intra- and inter-doublet INS transitions in the  $2(S=1/2)$  states as well as the intensities of the EPR transitions. The peculiarities of the INS and EPR spectra of the  $V_3$  ring of  $V_{15}$  quantum molecular magnet and EPR spectra of the  $V_3$  and  $Cu_3$  nanomagnets are described by the isosceles Heisenberg model with the DM exchange. Spin chirality and spin structure of the  $Cu_3$  and  $V_3$  nanomagnets with the Dzialoshinsky-Moriya (DM) exchange interaction are analyzed in the vector and scalar spin chirality models. The vector chirality model describes the field, orientation and deformation dependence of the spin chirality  $\kappa_n$ . The spin chirality is formed by the DM interaction and depends on the sign of the DM parameter  $G_z$ . The DM exchange and distortions determine the degree of chirality  $\kappa_n < 1$  in the isosceles clusters.

## 1. Introduction

Metal clusters have attracted significant interest as molecular magnets [1], possible components for molecule-based quantum computation [2-4], as well as active centers of biological systems [5]. In the equilateral  $Cu_3$  and  $V_3$  clusters, ( $J_{ij}=J_0$ ), the Heisenberg antiferromagnetic ( $J_0 > 0$ ) exchange interaction  $H_0 = \sum J_0 \mathbf{S}_i \mathbf{S}_j$  leads to the spin-frustrated ground state  $2(S=1/2)$  and excited  $S=3/2$  state [6, 7]. These trinuclear clusters are the simplest magnetic systems which allow one to investigate the effects of the Dzialoshinsky-Moriya [8, 9] (DM) exchange  $H_{DM} = \sum G_{ij} [\mathbf{S}_i \times \mathbf{S}_j]$  and distortions, anisotropy of magnetic and spectroscopic characteristics [6, 7], the spin-frustration, spin chirality, spin reorientation, and quantum magnetization. Large DM exchange in the  $Cu_3$  trimers with large  $J$  ( $> 150 \text{ cm}^{-1}$ ) was found and described in the DM(z) model with the DM parameters  $G_z = 5, 15-47 \text{ cm}^{-1}$  ( $G_z/J_{av} = 0.155-0.225$ ) [6, 7, 10-17]. For the equilateral clusters with large  $J_0$ , the DM exchange results in zero-field splitting (ZFS)  $2\Delta_{DM}^0 = |G_z| \sqrt{3}$  of the  $2(S=1/2)$  ground state (GS), and determines the anisotropy of magnetic and spectroscopic characteristics [6, 7, 10-17]. In the isosceles DM clusters with  $J_{13} = J_{23} \neq J_{12}$ , the ZFS  $2\Delta$  of the  $2(S=1/2)$  states is determined by  $G_z$  and  $\delta$ -distortion [6, 7]. The DM mixing of the spin states in the  $Cu_3$  clusters with large  $J$  and  $G$ , and the origin of the DM exchange parameters were considered in ref [18].

The DM exchange is also active in the clusters with small Heisenberg and DM parameters such as the  $\{Cu_3\}$  [19-21] and  $[V_3]$  [22] nanomagnets, as well as the  $V_3$  ring of  $V_{15}$  quantum molecular nanomagnet [23-33, 2a]. These trimers with small  $J$  ( $J = 1.7-3.4 \text{ cm}^{-1} = 2.4-4.8 \text{ K}$ ) have attracted much attention as molecular magnets [2a, 19-33]. The effect of quantum magnetization, owing to the spin-frustrated  $2(S=1/2)$  doublets, was observed first in the  $V_3$  ring of  $V_{15}$  [23, 24], and later in the  $[V_3]$  [22] and  $\{Cu_3\}$  [19, 20] nanomagnets. These clusters are characterized by the crossing of the  $|3/2, -3/2\rangle$  and  $|1/2, -1/2\rangle$  levels at level-crossing (LC) field  $H_{LC}$  ( $H_{LC} = 3J/2g\mu_B$ ) and tunneling gaps  $\Delta_{ij}$  at  $H_{LC}$ . The ZFS, tunneling gaps, quantum magnetization and EPR spectra of the  $V_3$  ring of  $V_{15}$  were explained in the equilateral DM model with the non-zero  $G_z$  and  $G_x, G_y$  parameters ( $G = 0.05-0.2 \text{ K}$ ) [23-33].

The microscopic origin of this  $2\Delta$ -gap of the spin-frustrated  $2(S=1/2)$  states of the  $V_3$  ring of  $V_{15}$  is a subject of discussion until now [33, 34]. The DM exchange coupling in the  $V_3$  ring of  $V_{15}$  was proposed [23-33, 3a] for the explanation of the  $2\Delta$ -gap and quantum magnetization. On the other hand, this  $2\Delta$ -gap was described by the isotropic pure Heisenberg scalene triangle model ( $J_{12} \neq J_{13} \neq J_{23}$ ) [34] on the basis of the observed inelastic neutron scattering (INS) spectra [34]. At the same time, recent EPR investigations [33] of the  $V_3$  ring of  $V_{15}$  show the angle dependence of the resonance fields, which was discussed in the equilateral DM exchange model [33]. The correlations between the INS and EPR spectra, chirality and geometry of the  $V_3$  clusters require the joint analysis of the INS and EPR spectra in the trimeric DM models. The influence of the DM exchange on the INS transitions was not considered in the Heisenberg spin models of the INS transitions [35-37].

The DM exchange, the ground state (GS) spin chirality and the tunneling gaps at LC field  $H_{LC}$ , play the principal role in explaining the quantum magnetization in the  $\{Cu_3\}$  [19, 20] and  $[V_3]$  [22] DM nanomagnets ( $G_n \approx 0.5 \text{ K}$ ). The spin chirality in the  $\{Cu_3\}$  DM nanomagnets was proposed as the parameter for electric control over a single molecular spin system which allows manipulation with the spin triangles as elements for molecule-based quantum computation [19-21]. However, the spin chirality of the  $Cu_3$  and  $V_3$  clusters with the DM exchange, the correlation between chirality and tunneling gaps, the dependence of spin chirality on magnetic field and distortions were not considered.

The aim of the paper is the consideration of i) the INS and EPR transitions in the  $V_3$  clusters with the DM exchange, and application of the DM exchange models for the explanation of the observed INS and EPR spectra of the  $V_3$  and  $Cu_3$  nanomagnets, and ii) the influence of the DM exchange on the spin chirality of the  $V_3$  and  $Cu_3$  nanomagnets, the field and deformation dependence of the spin chirality

## 2. The DM exchange splitting and mixing of spin states

The Hamiltonian of the distorted  $V_3$  and  $Cu_3$  clusters

$$H=(J_{12}S_1S_2+J_{23}S_2S_3+J_{13}S_1S_3)+H_{DM}+H_{ZFS}+\sum\mu_B\mathbf{S}_i\mathbf{g}_i\mathbf{H} \quad (1)$$

describes the isotropic Heisenberg exchange  $H_0$ , the DM exchange [8, 9]

$$H_{DM}=\sum G_{ij}[\mathbf{S}_i\times\mathbf{S}_j], \quad (2)$$

ZFS of the  $S=3/2$  state ( $H_{ZFS}=D_0[S_z^2-S(S+1)/3]$ ) and Zeeman interaction,  $ij=12, 23, 31$ .

In the equilateral cluster, the DM(z) coupling  $H_{DM}(z)=\Sigma G_{ij}z^L[\mathbf{S}_i\times\mathbf{S}_j]_{z^L}$  splits the spin-frustrated  $2(S=1/2)$  states on the two doublets with the energy  $E_{1,2}=-d_z, E_{3,4}=d_z; d_z=\frac{1}{2}G_z\sqrt{3}$  [6,7]. The spin eigenfunctions  $[u_+(-1/2), u_-(1/2)]$  and  $[u_+(-1/2), u_+(1/2)]$ , which diagonalize the  $H_{DM}(z)$  model in the representation  $\varphi_0, \varphi_1$  of the intermediate spins ( $S_{12}=0$  and 1 in  $\varphi_{S_{12}}(S, M)$ ) are the following:

$$u_+(-1/2)=|1, -1/2\rangle=-[\varphi_0(-1/2)+i\varphi_1(-1/2)]/\sqrt{2}=i[|\downarrow\downarrow\rangle+\omega|\uparrow\downarrow\rangle+\omega^2|\downarrow\uparrow\rangle]/\sqrt{3}, \quad (3)$$

$$u_-(1/2)=|-1, 1/2\rangle=[\varphi_0(1/2)-i\varphi_1(1/2)]/\sqrt{2}=-i[|\uparrow\uparrow\rangle+\omega|\uparrow\downarrow\rangle+\omega^2|\downarrow\uparrow\rangle]/\sqrt{3};$$

$$u_+(-1/2)=|1, -1/2\rangle=[\varphi_0(-1/2)-i\varphi_1(-1/2)]/\sqrt{2}=i[|\downarrow\downarrow\rangle+\omega^2|\uparrow\downarrow\rangle+\omega|\downarrow\uparrow\rangle]/\sqrt{3},$$

$$u_-(1/2)=|-1, 1/2\rangle=-[\varphi_0(1/2)+i\varphi_1(1/2)]/\sqrt{2}=-i[|\uparrow\uparrow\rangle+\omega^2|\uparrow\downarrow\rangle+\omega|\downarrow\uparrow\rangle]/\sqrt{3}.$$

$\omega=e^{2\pi i/3}$ , up and down arrows represent the up and down spins, respectively, for  $S_i$ . In the case of the existence of the  $G_x, G_y$  and  $G_z$  DM parameters [18b], the correlations between the in-plane components  $G_x, G_y$  of the DM vectors  $G_{ij}$  in the pair  $X_{ij}, Y_{ij}, Z_{ij}$  and the cluster  $X, Y, Z$  right-handed coordinate system have the form

$$G_{12,x}=G_{12,x_{12}}=G_x, G_{12,y}=G_{12,y_{12}}=G_y, G_{23,x}=-\frac{1}{2}(G_x+\sqrt{3}G_y), \\ G_{23,y}=\frac{1}{2}(\sqrt{3}G_x-G_y), G_{31,x}=-\frac{1}{2}(G_x-\sqrt{3}G_y), G_{31,y}=-\frac{1}{2}(\sqrt{3}G_x+G_y). \quad (4)$$

The pair DM parameters are equal in the equilateral system,  $G_{ij,x_{ij}}=G_x, G_{ij,y_{ij}}=G_y$ .

The Z components of the pair  $G_{ij}$  DM vector parameters are oriented perpendicular to the plain of the cluster  $G_{12,z}=G_{23,z}=G_{31,z}=G_z, Z_{ij}\parallel Z$ . The in-plane ( $G_x, G_y$ ) DM exchange results in the mixing of the  $S=1/2$  and  $S=3/2$  states [25, 27-33, 18-22], which plays significant role in the  $V_3$  and  $Cu_3$  nanomagnets [18-33]. The group-theoretical analysis of the DM mixing in the  $V_3$  ring of  $V_{15}$  was considered in refs [30, 31]. The matrix elements of the DM exchange mixing of the  $S=1/2$  and  $\Phi(3/2)$  states have the form

$$\langle u_{\pm}(\pm 1/2) | \Phi(\pm 3/2) \rangle = (3i\sqrt{2}/4)G_{\pm}, \langle u_{\pm}(\mp 1/2) | \Phi(\pm 1/2) \rangle = (i\sqrt{6}/4)G_{\pm}, \\ \langle u_{\pm}(\mp 1/2) | \Phi(\mp 3/2) \rangle = 0, \quad \langle u_{\pm}(\pm 1/2) | \Phi(\mp 1/2) \rangle = 0. \quad (5)$$

where  $G_{\pm}=(G_x \pm iG_y)/\sqrt{2}$ . The energy levels of the equilateral [ $V_3$ ] cluster ( $J=4.8K, G_z=-0.5K, G_x=0.5K, G_y=0$ ), the spin chirality, ZFS, and the DM mixing are shown in Fig.1,  $H=H_z$ . The lowest zero-field (ZF) state for  $G_z<0$  is the  $[u_+(-1/2), u_-(1/2)]$  doublet. The DM exchange ( $G_x$ ) results in the tunneling gap  $\Delta_{12}$  in the ground branch at LC field  $H_{LC1}$  and simple level crossing ( $\Delta_{23}=0$ ) in the excited state at  $H_{LC2}$  for  $G_z<0$ , Fig. 1.

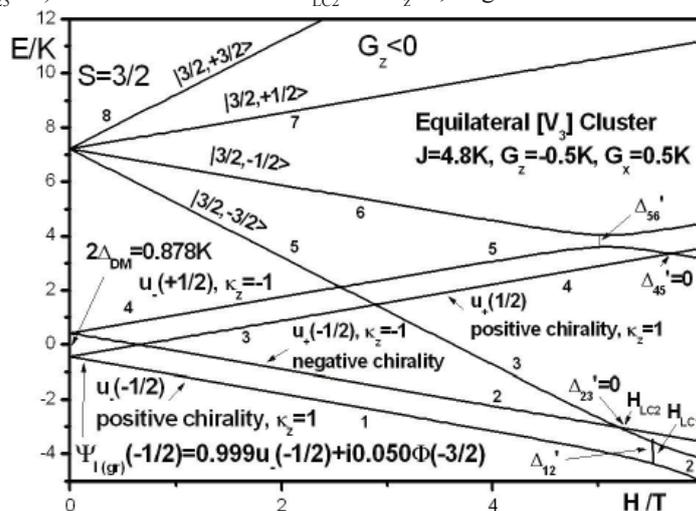


Fig. 1. The energy levels scheme, spin chirality, the DM exchange mixing and tunneling gaps in the equilateral  $V_3$  cluster.  $J=4.8K, G_z=-0.5K, G_x=0.5K, G_y=0$ .

The isosceles  $\{\text{Cu}_3\}$  nanomagnets [19, 20] with small  $J$  ( $J_{12} = 4.52$  K,  $J_{13} = J_{23} = 4.04$  K) were characterized by the strong in-plane and out-of-plane DM( $x,y,z$ ) exchange coupling  $|G_z| = G_x = G_y = 0.53$  K;  $G_z/J_{av} = 0.126$ ,  $G/J_{av} = 0.218$  [20]. Fig. 2 shows the energy levels scheme, tunneling gaps, the INS and EPR transitions for this  $\text{Cu}_3$  nanomagnet with different  $G_z$  parameters:  $G_z = +0.53$  K (dashed lines) and  $G_z = -0.53$  K (solid lines),  $g_{av} = 2.06$ . For  $H < 2$  T, the splittings do not depend on the sign of  $G_z$ . The in-plane ( $G_x, G_y$ ) DM spin mixing results in the large tunneling gap  $\Delta_{12}'$  in the ground branch at LC field  $H_{LC1}$  and small tunneling gap  $\Delta_{23}'$  at  $H_{LC2}$  in the excited state for  $G_z < 0$ , Fig. 2, solid. In the case  $G_z > 0$ , small tunneling gap  $\Delta_{12}'$  in the ground branch at LC field  $H_{LC1}$  and large tunneling gap  $\Delta_{23}'$  in the excited branch at  $H_{LC2}$  take place, Fig. 2, dash.

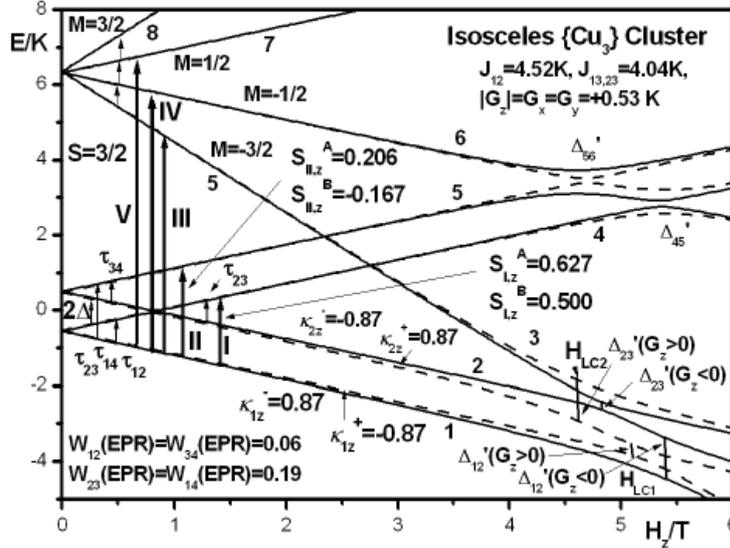


Fig. 2. Spin levels, the INS and EPR transitions, tunneling gaps in the isosceles  $\text{Cu}_3$  nanomagnet.

### 3. Intensities of INS transitions in isosceles clusters with DM exchange

The expressions for the differential magnetic cross-section of INS, the intensities of the INS transitions in the Heisenberg clusters were presented in refs [34-37]. The INS transitions are determined by the spin structure factors  $S_N(\mathbf{Q})$  [36], where  $\mathbf{Q}$  is the scattering vector. The scheme of the INS transitions for the  $V_3$  and  $\text{Cu}_3$  isosceles nanomagnets is shown in Fig. 2. The analysis [34] of the observed INS transitions in the  $V_3$  ring of  $V_{15}$  in the scalene Heisenberg model ( $G_n = 0$ ) results in the intensity ratios (III:IV:V)=3:2:1 for the transitions III, IV and V (Fig. 2). The Q-dependence of these transitions was described [34] very well by the equations  $I_{III} = 1/2[1 - \sin(QR)/QR]$ ,  $I_{IV} = 1/3[1 - \sin(QR)/QR]$ ,  $I_V = 1/6[1 - \sin(QR)/QR]$ ,  $I_{III} + I_{IV} + I_V \sim [1 - \sin(QR)/QR]$ . For description of the Q-dependent intensity of the intra-doublet INS transition I (Fig. 2) of a scalene trimer with ground state  $\Omega_0(\pm 1/2) = a\phi_0(\pm 1/2) + b\phi_1(\pm 1/2)$ , the equation  $I_I = I_0 F^2(Q) [a^2 + 1/3 b^2 (1 - \sin(QR)/QR)]$  was proposed [34].

The consideration of the spin structure factors shows that the scalene Heisenberg model ( $G_n = 0$ ) cannot describe the Q-dependence of the INS transition I [39]. The analysis of the INS and EPR spectra requires the taking in account the DM exchange.

The calculations of the INS for the isosceles DM trimer result in the structure factors for the INS transitions I-V (Fig. 2) in magnetic field  $H = H_z$

$$\begin{aligned} S_I' &= 1/2 + (G_z^2/8\Delta^2) [1 - 4\cos(\mathbf{Q}\mathbf{R}_{23})], \\ S_{II}' &= 1/3 [1 - \cos(\mathbf{Q}\mathbf{R}_{12})] - (G_z^2/8\Delta^2) [1 - 4\cos(\mathbf{Q}\mathbf{R}_{23})], \\ S_{III}' &= 1/2 [1 - \cos(\mathbf{Q}\mathbf{R}_{12})], S_{IV}' = 1/3 [1 - \cos(\mathbf{Q}\mathbf{R}_{12})], S_V' = 1/6 [1 - \cos(\mathbf{Q}\mathbf{R}_{12})], \end{aligned} \quad (6)$$

$\Delta = (\delta^2 + d_z^2)^{1/2} = 1/2[(J_{12} - J_{23})^2 + 3G_z^2]^{1/2}$ . The structure factors for the transitions I and II at high field  $H \perp Z$  are reduced to their values in the pure Heisenberg model, since high transverse magnetic field  $H_{\perp}$  suppresses the effect of the DM exchange. The average structure factors  $S_{N,av}' = S_N'$  for the INS transitions in the IS trimer have the form:

$$\begin{aligned} S_I' &= 1/2 + (G_z^2/24\Delta^2) [1 - 4\sin(QR)/QR], \\ S_{II}' &= 1/3 [1 - \sin(QR)/QR] - (G_z^2/24\Delta^2) [1 - 4\sin(QR)/QR], \\ S_{III}' &= 1/2 [1 - \sin(QR)/QR], S_{IV}' = 1/3 [1 - \sin(QR)/QR], S_V' = 1/6 [1 - \sin(QR)/QR]. \end{aligned} \quad (7)$$

In the absence of the DM exchange ( $G_z = 0$ ), the structure factor  $S_I'$  (7) of the INS transition I (Fig. 2) is reduced to the Q-independent form  $S_I' = 1/2$ ; the structure factor  $S_{II}'$  (7) of the INS transition II is reduced to  $S_{II}' = 1/3 [1 - \sin(QR)/QR]$ , the structure factors of the Heisenberg isosceles trimer. Eq (7) shows significant influence of the DM exchange on the intensities of the intra-doublet transition I ( $S_I' = [S_I^A - S_I^B \sin(QR)/QR]$ ) and doublet-doublet transition II ( $S_{II}' = [S_{II}^A - S_{II}^B \sin(QR)/QR]$ ). Thus, the  $S_I^B$  term of the DM exchange origin  $S_I^B = \Delta S_I^B = (G_z^2/6\Delta^2)$  (7) results in the Q-dependence

of the transition I [ $S_I^B=S_{1,0}^B+\Delta S_I^B$ ,  $S_{1,0}^B=0$ ]. The DM exchange switches on and increases the Q-dependence of the INS transition I and, at the same time, decreases the Q-dependence of transition II. The redistribution of the intensities of the INS transitions II and I, which is controlled by the DM exchange and distortions (the  $[G_z/\Delta]^2$  term in (7)), takes place with the conservation rule  $S_I^A+S_{II}^A=S_{1,0}^A+S_{II,0}^A$  for the Q-independent terms, and  $S_I^B+S_{II}^B=S_{1,0}^B+S_{II,0}^B$  for the Q-dependent terms.

Calculated values of the  $S_I^A$ ,  $S_{II}^A$  and  $S_I^B$ ,  $S_{II}^B$  coefficients of the Q-independent and Q-dependent terms, respectively, of the averaged structure factors  $S_I^A=S_I^A-S_I^B\sin(QR)/QR$  and  $S_{II}^A=S_{II}^A-S_{II}^B\sin(QR)/QR$  of the INS transitions I and II, are shown in Fig. 2 for the set of the Heisenberg and DM parameters of the  $Cu_3$  nanomagnet, which were determined [19, 20] in the magnetization and EPR experiment. The values of the structure factors in Fig. 2 [ $S_{Iz}^A=0.627$  ( $S_{1,0}^A=0.5$ ),  $S_{Iz}^B=0.500$  ( $S_{1,0}^B=0$ );  $S_{IIz}^A=0.206$  ( $S_{II,0}^A=0.333$ ),  $S_{IIz}^B=-0.167$  ( $S_{II,0}^B=+0.333$ )] show significant influence of the DM exchange on the intensities of the INS transitions.

The Q-dependence of the transitions I and II allows one to experimentally determine the  $|G_z/2\Delta|$  relation. Thus, the experimentally observed Q-dependence of the transition I in the  $V_3$  ring of  $V_{15}$  was described [34] by the Q-independent term  $(a^2+b^2/3)=0.6$  and Q-dependent term  $[-0.2\sin(QR)/QR]$  [34]. For case, where the Q-dependence of the INS transition I is determined by Eq (7), the comparison with the coefficient  $[(G_z/\Delta)^2/6]$  in the Q-dependent term in  $S_I^A$  (7) leads to the estimate  $G_z/2\Delta\approx 0.55$ . Since  $2\Delta\approx 0.31K$  and  $J_{av}=2.46K$  [34], this estimate results in  $G_z\approx 0.17K$  and  $\delta=0.06K$ . In this case, the Q-independent term in the structure factor  $S_I^A$  (7) of transition I is  $S_I^A\approx 0.55$ . This value is close to the Q-independent term 0.6 for I in [34], that allows one to explain qualitatively the observation [34] that the overall intensity of peak I is significantly smaller than the sum of (III+IV+V).

In the isosceles [ $V_3$ ] clusters, the DM exchange results in i) the Q-dependence of the spin structure factor  $S_I^B$  of the INS intra-doublet transition I (the coefficient  $S_I^B$  in  $S_I$ ) and ii) the redistribution of the Q-independent  $S_I^A$  and  $S_{II}^A$  parts, as well as the Q-dependent,  $S_I^B$  and  $S_{II}^B$ , parts of the intensities of the INS transitions I and II with the conservation of the summary intensities of these two transitions:  $S_I^A+S_{II}^A=S_{1,0}^A+S_{II,0}^A=5/6$   $\{S_I^B+S_{II}^B=S_{1,0}^B+S_{II,0}^B=1/3\}$ .

#### 4. EPR transitions in isosceles clusters with DM exchange

In the pure Heisenberg isosceles model, only intra-doublet  $1\rightarrow 2(3)$  and  $3(2)\rightarrow 4$  EPR transitions are allowed for  $H=H_z\parallel Z$  and  $H(\perp Z)=H_\perp$  ( $W_{1\rightarrow 2(3)}=W_{3(2)\rightarrow 4}=0.25$ ,  $W_{14}=W_{23}=0$ ). For the isosceles trimer with the DM exchange, the relative intensities of the allowed EPR transitions ( $\tau_{13}$ ,  $\tau_{24}$ ,  $\tau_{23}$ ,  $\tau_{14}$  in Fig. 2,  $\hbar\nu>2\Delta$ ) for  $H=H_z$  are determined by the equation  $W_{13}=W_{24}=\delta^2/4\Delta^2$ ;  $W_{14}=W_{23}=d_z^2/4\Delta^2$  [6, 7]. At high transverse magnetic field,  $h_x\gg\Delta$ , the effect of the DM exchange is suppressed:  $W_{13,x}=W_{24,x}=0.25$ ,  $W_{14,x}=W_{23,x}=0$ . Fig. 3 shows the frequency dependences  $[(\nu_{ij}/\gamma_g)-H]$  of the resonance fields for the  $2(S=1/2)$  states of the isosceles trimer with the DM exchange,  $\gamma_g=g\mu_B/\hbar$ .

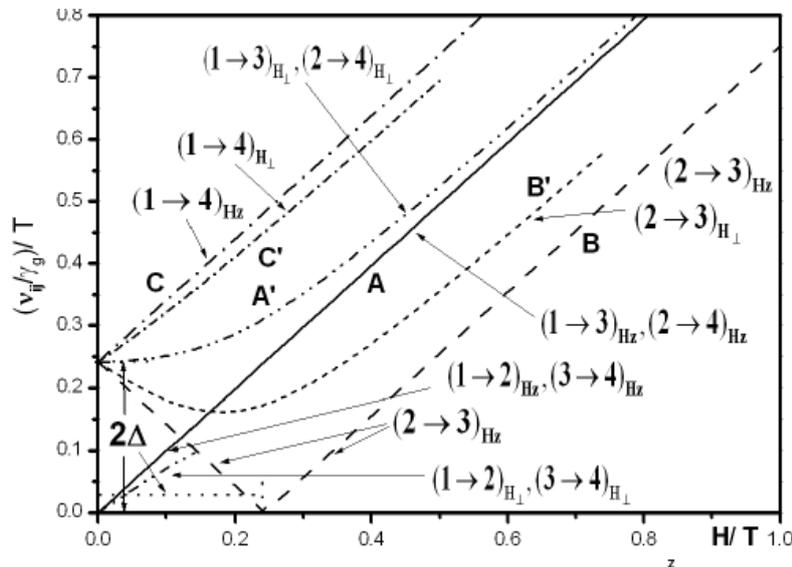


Fig. 3 Frequency  $(\nu/\gamma_g)$  –field (H) diagram for  $H_z\parallel Z$  and  $H_x\perp Z$ .

The straight A (solid), B (dash), and C (dash-dot) lines show the resonance conditions for the transitions  $(1\rightarrow 3)_{H_z}$ ,  $(2\rightarrow 4)_{H_z}$  [ $W_{13,H_z}=W_{24,H_z}=0.14$ ] for  $\hbar\nu>2\Delta_{SC}$   $\{(1\rightarrow 2)_{H_z}$ ,  $(3\rightarrow 4)_{H_z}$  (for  $\hbar\nu<2\Delta_{SC})\}$ ,  $(2\rightarrow 3)_{H_z}$  and  $(1\rightarrow 4)_{H_z}$ , [ $W_{23,H_z}=W_{14,H_z}=0.11$ ], respectively, at magnetic field  $H_z\parallel Z$ . The A' (dash-dot-dot) {B' (shot dash)} [C' (shot dash-dot)] curve shows the resonance conditions for the resonance fields for the EPR transitions  $(1\rightarrow 3)_{H_x}$ ,  $(2\rightarrow 4)_{H_x}$   $\{(2\rightarrow 3)_{H_x}\}$  [ $(1\rightarrow 4)_{H_x}$ ] at magnetic field  $H\perp Z$ . The non-linear field dependence A', B' (Fig. 3) of the frequency dependences  $\nu(H)$  of the resonance fields at magnetic field  $H\perp Z$  is characteristic for the DM exchange in the trimer. The low-frequency

EPR spectra [33] of the  $V_3$  ring of  $V_{15}$  quantum molecular magnet were explained by the authors [33] in the equilateral DM model. The inter-doublet EPR transitions  $1 \rightarrow 4$  and  $2 \rightarrow 3$  and the intra-doublet  $1 \rightarrow 3$  and  $2 \rightarrow 4$  transitions with weak intensity for  $H=H_z$  were observed in [33] (Fig. 2b[33]), as well as the linear magnetic behavior of the resonance frequencies for  $H\parallel Z$  and non-linear magnetic behavior at  $H\perp Z$  (Fig. 2a [33]) induced by the DM exchange. The observation of the inter-doublet  $(1 \rightarrow 4)_z$  and  $(2 \rightarrow 3)_z$  EPR transitions and linear {non-linear} v-H magnetic behavior for  $H\parallel Z$  { $H\perp Z$ } is the evidence of the presence of the DM exchange in  $V_3$  ring (Fig. 2, B,C {A'}). At the same time, the intra-doublet low-frequencies  $(1 \rightarrow 3)_z$  and  $(2 \rightarrow 4)_z$  transitions are forbidden for the equilateral DM model and are allowed for the isosceles  $V_3$  ring, Figs. 2, 3 ( $W_{13}=W_{24}=\delta^2/4\Delta^2$ ). The observation of the  $\tau_{13z}, \tau_{24z}$  as well as the  $\tau_{14z}, \tau_{23z}$  EPR transitions on the  $V_3$  ring of  $V_{15}$  [33] shows that this  $V_3$  ring has the symmetry of the isosceles triangle (not equilateral) with the DM exchange. The observed correlation ( $W_{13} \approx W_{24} < W_{14} \approx W_{23}$ ) between the intensities of the EPR transitions [33] corresponds to the relation  $\delta < G_z \sqrt{3}$  in the isosceles DM model. The analysis of the calculated INS and EPR transitions [39] for the equilateral  $V_3$  cluster and comparison with the observed INS [34] and EPR [33] transitions shows that the equilateral DM model cannot describe the EPR and INS spectra of the  $V_3$  ring of  $V_{15}$ , the isosceles  $\delta$ -distortion should be included in the consideration. The EPR spectra of the  $V_3$  [22] and  $Cu_3$  [19, 20] nanomagnet also are described in the isosceles DM model.

### 5. Spin chirality of the $Cu_3$ and $V_3$ nanomagnets with DM exchange

Recently, the spin chirality of the  $\{Cu_3\}$  DM nanomagnet was proposed as the parameter for the manipulation with the spin triangles as units for molecule-based quantum gates [19-21]. The spin chirality of the magnetic systems is usually considered in the scalar chirality model and in the vector chirality model. The spin chirality in the  $\{Cu_3\}$  nanomagnet was considered [21], using the scalar chirality operator  $C_z$

$$C_z = (4/\sqrt{3})\mathbf{S}_1 \cdot [\mathbf{S}_2 \times \mathbf{S}_3]. \quad (8)$$

The matrix elements of  $C_z$  in the  $u_{\pm}(\pm 1/2)$  basis (3) have the form

$$\chi_+ = \langle u_+(\pm 1/2) | \hat{C}_z | u_+(\pm 1/2) \rangle = 1, \quad \chi_- = \langle u_-(\pm 1/2) | \hat{C}_z | u_-(\pm 1/2) \rangle = -1. \quad (9)$$

The operator  $C_z$  splits the  $2(S=1/2)$  set on the states  $u_+(M_S)$  and  $u_-(M_S)$  characterized by the projections  $M_L = \pm 1$  of the pseudoorbital moment  $L$  and does not act on the spin moments  $M_S, M_S = \pm 1/2$ . The scalar chirality  $\chi = \pm 1$  pseudospin coincides with  $M_L = \pm 1$ .

In the case of the vector chirality [38] which can be defined for the  $S_i=1/2$  trimer as

$$\mathbf{K}_z = (2/\sqrt{3})\{[\mathbf{S}_1 \times \mathbf{S}_2]_z + [\mathbf{S}_2 \times \mathbf{S}_3]_z + [\mathbf{S}_3 \times \mathbf{S}_1]_z\}, \quad (10)$$

the chirality vector  $\mathbf{K}_z$  is parallel to  $Z$ -axis with amplitude  $+1$  or  $-1$ , since the matrix elements of  $\mathbf{K}_z$  in the  $u_{\pm}(\pm 1/2)$  basis have the form

$$\kappa_z = \langle u_{\pm}(\pm 1/2) | \hat{\mathbf{K}}_z | u_{\pm}(\pm 1/2) \rangle = 1, \quad \kappa_z = \langle u_{\pm}(\mp 1/2) | \hat{\mathbf{K}}_z | u_{\pm}(\mp 1/2) \rangle = -1. \quad (11)$$

The chirality is the sign of the projection of the spin vector onto the orbital momentum vector: negative is left, positive is right. In the positive (right) chiral states  $u_+(1/2)$  and  $u_-(-1/2)$  with  $\kappa_z = +1$  (11), the direction of the spin moment ( $M_S$ ) coincides with the direction of the pseudoorbital moment ( $M_L$ ): thus,  $M_L = -1, M_S = -1/2$ , and the total pseudoangular moment is  $M_J = M_L + M_S = -3/2$  for  $u_-(-1/2)$ ;  $M_L = +1, M_S = +1/2, M_J = M_L + M_S = 3/2$  for  $u_+(1/2)$ . In the negative (left) chiral states  $u_+(-1/2)$  and  $u_-(1/2)$ ,  $\kappa_z = -1$ , the directions of  $M_S$  and  $M_L$  are opposite: thus,  $M_L = -1, M_S = 1/2, M_J = -1/2$  for  $u_-(1/2)$ ; for  $u_+(-1/2) - M_L = +1, M_S = -1/2, M_J = +1/2$ . The two states with  $M = -1/2$  in Fig. 1 possess different vector spin chirality: In the case  $G_z < 0$ , the GS is the positive (right) chiral state  $u_-(-1/2)$ ,  $\kappa_z = +1, M_L = \chi_{1z} = -1$ , which exhibits the in-plane ( $G_x, G_y$ ) DM exchange repulsion from the  $|3/2, -3/2\rangle$  state that results in the tunneling gap  $\Delta_{12}$  at LC field  $H_{LC1}$  in the ground branch, Fig. 1. The first excited negative chiral state  $u_+(-1/2)$ ,  $\kappa_z = -1$  does not exhibit the DM mixing with the  $|3/2, -3/2\rangle$  state, that results in the simple crossing ( $\Delta_{23} = 0$ ) at  $H_{LC2}$ , Fig. 1. The equilateral trimers with  $G_z > 0$  and left chiral GS  $u_+(-1/2)$ ,  $\kappa_z = -1, \chi_{1z} = +1$ , possess the simple crossing ( $\Delta_{12} = 0$ ) at LC field  $H_{LC1}$  in the ground branch and the tunneling gap  $\Delta_{23}$  at LC field  $H_{LC2}$  in the excited state. These correlations are consistent with the results of the group-theoretical analysis [30].

The DM exchange  $H_{DM}$  forms the chiral states of the DM trimer, the sign of  $G_z$  determines the vector spin chirality  $\kappa_z$  of the ground and excited states. The spin chirality of the pure Heisenberg states ( $G_i = 0$ ) is equal to zero.

Fig. 4 shows the spin chirality  $\kappa_1$  of the ground state and  $\kappa_2$  of the first excited state of the equilateral  $V_3$  cluster with the exchange parameters  $J_0 = 4.8K, G_z = \pm 0.5K, G_{xy} = 0$ , in magnetic field  $H = H_z \parallel Z$  ( $\kappa_{1z}$ ) and  $H = H_x \perp Z$  ( $\kappa_{1x}, \kappa_{2x}$ ).  $\kappa_{1x}^+$  and  $\kappa_{1x}^-$  correspond to  $G_z > 0$  and  $G_z < 0$ , respectively. The spin chirality in the DM(z) model (Fig. 4, short-dash-dot) does not depend on  $H_z$  for  $H_z < H_{LC}$ : i)  $\kappa_{1z,0} = +1$  for the positive chiral ground state  $u_-(-1/2)$ ,  $G_z < 0$ , and ii)  $\kappa_{1z,0} = -1$  for the negative chiral ground state  $u_+(-1/2)$ ,  $G_z > 0$ .

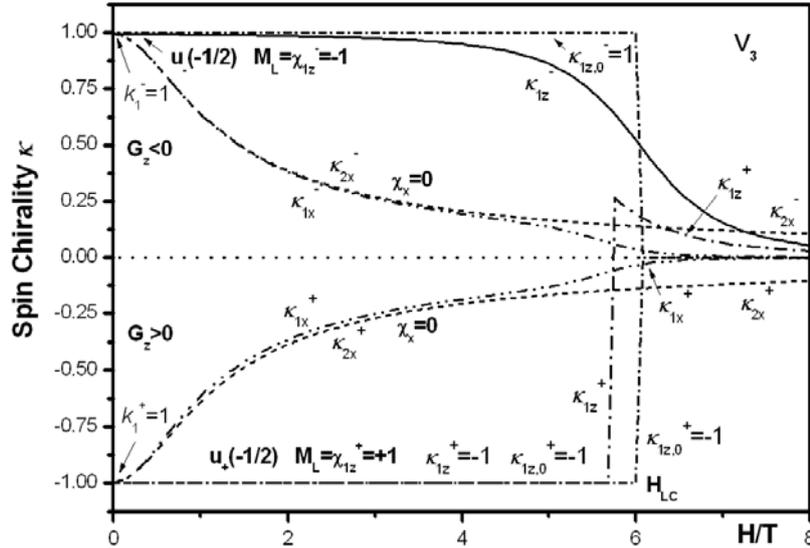


Fig. 4 Field dependence of the spin chirality of the equilateral  $V_3$  cluster.

Chirality  $\kappa_{1z,0}$  changes sharply to the value  $\kappa_{1z,0}=0$  ( $M_{gr}=-3/2$ ) at LC field  $H=H_{LC}$ . Fig. 4 also shows the field dependence of the spin chirality of the  $V_3$  cluster ( $J_0=4.8K$ ,  $G_z=\pm 0.5K$ ) with the in-plane ( $G_x=0.5K$ ) DM mixing. Positive chirality  $\kappa_{1z}=-1$  of the ground  $u(-1/2)$  state ( $G_z<0$ ) changes smoothly to the value  $\kappa_{1z}=0$  after the avoided crossing (Fig. 4) due to the  $G_x$  DM mixing in accord with Fig. 1. In the case  $G_z>0$ ,  $G_x\neq 0$ , negative chirality  $\kappa_{1z}^+=-1$  of the ground  $u(-1/2)$  state is field-independent for  $H_z<H_{LC}$  and then  $\kappa_{1z}^+$  changes abruptly at  $H_{LC}$  (Fig. 4) due to the simple level crossing. The states of the different vector chirality  $\kappa$  are characterized by the same scalar  $\chi$  pseudospin:  $u(1/2)$ ,  $\kappa_z=-1$ ,  $\chi=-1$  and  $u(-1/2)$ ,  $\kappa_z=+1$ ,  $\chi=-1$ .

In the case of the transverse field  $H_x \perp Z$ , the vector chirality operator  $K_z$  describes the projection of the spin chirality vector on the magnetic field. Eq (12) describes the field dependence  $\kappa_{1x}(H_x)$  and  $\kappa_{2x}(H_x)$  of the vector spin chirality of the ground and first excited states

$$\kappa_{1x} = \kappa_{2x} = \pm |d_z| / \sqrt{d_z^2 + h_x^2}, \quad (12)$$

the sign + (-) corresponds to  $G_z<0$  ( $G_z>0$ ),  $h_x=1/2g\mu_B H_x$ , Fig. 4. In the case  $G_z<0$ , the ZF chirality of the ground and first excited states (10) is positive and equal to 1:  $\kappa_{1x}^-=\kappa_{2x}^-=1$ , as in the field  $H_z$ , Fig. 4. In the case  $G_z>0$ , the ZF chirality is negative  $\kappa_{1x}^+=\kappa_{2x}^+=-1$ , Fig. 4. In the DM(z) model, the vector spin chirality  $\kappa_{1x}(H_x)$  of the ground state changes abruptly to the value  $\kappa_{1x}=0$  at  $H_{LC}$  since  $\kappa(S=3/2)=0$ .

In the transverse magnetic field  $H_x$ , the scalar chirality is equal to zero,  $\chi=0$ .

In the isosceles trimer (Fig. 2), the vector spin chirality for the states with  $M=-1/2$  in magnetic field  $H=H_z$  has the form  $\kappa_z = |d_z|/\Delta$ ,  $\kappa_z' = -|d_z|/\Delta$ ,  $\Delta=(\delta^2+d_z^2)^{1/2}$ ,  $\delta=1/2(J_{12}-J_{23})$ . The Heisenberg  $\delta$ -distortion reduces the exchange symmetry of the system, destroys the spin chirality and, together with  $d_z$ , determines the degree  $\kappa_z=|d_z|/\Delta$  of the positive [negative] chirality of the states of the isosceles DM clusters. For  $G_z<0$ , the two lowest states with  $M=-1/2$  in Fig 2 are characterized by the positive  $\kappa_{1z}=0.87$  and negative  $\kappa_{2z}=-0.87$  vector chirality, respectively. The dominant positive chiral GS ( $G_z<0$ ) in Fig. 2 corresponds to the large tunneling gap  $\Delta_{12}'$  in the ground branch at  $H_{LC1}$  and small gap  $\Delta_{23}'$  in the excited branch at  $H_{LC2}$ . The dominant negative chiral GS ( $G_z>0$ ) corresponds to small tunneling gap  $\Delta_{12}'$  in the ground branch at  $H_{LC1}$  and large gap  $\Delta_{23}'$  in the excited branch at  $H_{LC2}$ . Since the intensities of the inter- and intra-doublet EPR transitions in Fig. 2 have the form  $W_{14}=W_{23}=(d_z/2\Delta)^2$ ,  $W_{12}=W_{34}=(\delta/2\Delta)^2$ , there is a direct correlation between the spin chirality  $\kappa_z$  in the isosceles cluster, on the one side, and the intensities of the EPR and INS (see Eqs (6), (7)) transitions, on the other side,

$$W_{14}=W_{23}=\kappa_z^2/4; W_{12}=W_{34}=(1-\kappa_z^2)/4. \quad (13)$$

$$S_I = 1/2 + (\kappa_z^2/6)[1 - 4\cos(QR_{23})], \quad (14)$$

$$S_{II} = 1/3[1 - \cos(QR_{12})] - (\kappa_z^2/6)[1 - 4\cos(QR_{23})];$$

$$S_I' = 1/2 + (\kappa_z^2/18)[1 - 4\sin(QR)/QR], S_{II}' = 1/3[1 - \sin(QR)/QR] - (\kappa_z^2/18)[1 - 4\sin(QR)/QR].$$

The degree of chirality  $\kappa_z=|d_z|/\Delta$  may be determined from the EPR and INS experiments. The scalar chirality  $\chi(H_z)$  of the  $M=-1/2$  state in Fig.4 have the opposite sign in comparison with  $\kappa_z$ ,  $\chi_{1z}=-|d_z|/\Delta$ ,  $\chi_{2z}=d_z/\Delta$ .

In the case of the transverse magnetic field,  $H=H_x$ , the field and deformation dependence of the vector chirality of the isosceles  $Cu_3$  cluster has the form

$$\kappa_{1x} = |d_z| / [(\delta+h_x)^2 + d_z^2]^{1/2}, \quad \kappa_{2x} = |d_z| / [(\delta-h_x)^2 + d_z^2]^{1/2}. \quad (15)$$

The vector chirality correlate with the intensities of the inter-doublet EPR transitions  $(1 \rightarrow 4)_x$  and  $(2 \rightarrow 3)_x$  in the transverse field  $H = H_x$ ,

$$W'_{14}(H_{\perp}) = \kappa_{1x}^2 / 4, \quad W'_{23}(H_{\perp}) = \kappa_{2x}^2 / 4. \quad (16)$$

The scalar chirality in the field  $H_{\perp}$  is equal to zero.

The operator of the vector chirality  $\mathbf{K}_z$  describes the spin chirality of the  $S=1/2$  states in the  $\text{Cu}_3$  and  $\text{V}_3$  nanomagnets, its field, orientation and distortion dependence. The operator of the scalar chirality  $C_z$  describes the pseudoorbital moment  $\chi = M_L$ ;  $\chi = 0$  in the transverse field  $H_{\perp}$ .

## 6. Conclusion

The DM exchange results in i) the Q-dependence of the structure factor  $S_1$  of the INS intra-doublet transition I and ii) redistribution of the Q-independent (Q-dependent) parts of the intensities factors of the intra-doublet I and doublet-doublet II INS transitions with the conservation of the summary Q-independent (Q-dependent) intensities of these two transitions. For the intra-doublet and doublet-doublet transitions, the changes and redistribution of the intensities of the INS transitions I and II, on the one hand, and the intensities of the EPR transitions, on the other hand, have the same origin: they are controlled by the DM exchange and distortions (the  $[G_z/\Delta]^2$  terms). The joint consideration of the INS and EPR transitions in the  $\text{V}_3$  clusters in the Heisenberg plus DM exchange models shows that the Q-dependence of the INS transitions, peak positions and EPR transitions in the  $\text{V}_3$  ring of  $\text{V}_{15}$  quantum molecular magnet as well as EPR transitions in the  $\text{V}_3$  and  $\text{Cu}_3$  nanomagnets can be explained in the isosceles model with the DM exchange.

The vector chirality model describes the field, deformation and orientation dependence of the spin chirality  $\kappa_n$  of the  $\text{Cu}_3$  and  $\text{V}_3$  nanomagnets with DM exchange. The spin chirality is formed by the DM interaction, depends on the sign of the DM parameter  $G_z$  and is equal to zero for the pure Heisenberg clusters. The DM exchange and distortions determine the degree of chirality  $\kappa_n < 1$  in the isosceles clusters. The spin chirality  $\kappa_n$  correlates with the intensities of the EPR and INS transitions.

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